Week 6

Scaling laws

CS324
Motivating problem: hyperparameter costs

Hyperparameter tuning is a huge cost!

How can we solve this?
1. Guess and pray
2. Exhaustive search
3. Have simple rules that find optimal hyperparams

<table>
<thead>
<tr>
<th>Consumption</th>
<th>CO\textsubscript{2}e (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air travel, 1 passenger, NY→SF</td>
<td>1984</td>
</tr>
<tr>
<td>Human life, avg, 1 year</td>
<td>11,023</td>
</tr>
<tr>
<td>American life, avg, 1 year</td>
<td>36,156</td>
</tr>
<tr>
<td>Car, avg incl. fuel, 1 lifetime</td>
<td>126,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Training one model (GPU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLP pipeline (parsing, SRL)</td>
</tr>
<tr>
<td>w/ tuning &amp; experimentation</td>
</tr>
<tr>
<td>Transformer (big)</td>
</tr>
<tr>
<td>w/ neural architecture search</td>
</tr>
</tbody>
</table>

Table 1: Estimated CO\textsubscript{2} emissions from training common NLP models, compared to familiar consumption.\textsuperscript{1}

Strubell+ 2019
Teaser: simple, predictive ‘laws’ for behaviors of LMs

What you’ll learn today:

**scaling laws** which are simple, predictive rules for model performance

**Old and unpleasant**: tune hyperparameters on big models

**New and exciting**: tune on small models, extrapolate to large ones
Scaling laws: surprisingly clean and robust

These scaling laws hold on *many* different kind of phenomena!

They even hold in non-standard settings (when train ≠ test)
All you want to know about scaling laws (and more)

Organization: simple to complex

1. Data vs performance
   “Are there simple rules that determine how data affects performance?”

2. Hyper-parameters vs performance
   “Are optimal hyperparameters the same across different data/models?”

3. Forecasting with scaling laws
   “Does benchmark performance follow predictable trends?”
Data vs performance

What’s a data scaling law?

Data scaling laws: simple formula that maps dataset size \( n \) to error

What do we expect out of scaling laws?

Monotonic, logistic-like curves

[Hestness+ 2017]
Data scaling laws for language models

First, an empirical observation

Loss and dataset size is linear on a log-log plot

“Scale-free” or “Power law”

(For language modeling, from Kaplan+ 2020)
Scaling laws: past works and other areas

Scaling laws hold in many domains

Machine translation
Speech
Language modeling
Object recognition

Kaplan et al. 2020.
Rosenfeld 2020.

Data scaling has been known for a while
Kolachina+ 2012 for machine translation, Hestness+ 2017 for neural
Conceptual foundations of data scaling laws.

**Q:** Why do scaling laws show up?

We know error should be monotone

But why is it a power law / linear in log-log?

**A:** Estimation error naturally decays polynomially.

But this answer may take a moment to understand. Let’s work through an example.

**Example:** If our task is to estimate the mean of a dataset, what’s the scaling law?
Toy example: mean estimation

**Input:** $x_1 \ldots x_n \sim N(\mu, \sigma^2)$

**Task:** estimate the average as $\hat{\mu} = \frac{\sum_i x_i}{n}$

**What’s the error?** By standard arguments..

$$E[(\hat{\mu} - \mu)^2] = \frac{\sigma^2}{n}$$

**This is a scaling law!!**

$$\log(\text{Error}) = -\log n + 2 \log \sigma$$

More generally, any polynomial rate $1/n^\alpha$ is a scaling law
Fact: Similar arguments show most ‘classical’ models (regression, etc) have $\frac{1}{n}$ scaling

This means we should see $y = -x + C$

What do we find in neural scaling laws?

Very different from predictions.. Why might this be?
Detour: scaling laws for (nonparametric) learning

Neural nets can approximate arbitrary functions. Let’s turn that into an example.

**Input:** $x_1 \ldots x_n$ uniform in 2D unit box. $y_i = f(x_i) + N(0,1)$

**Task:** estimate $f(x)$

**Approach:** cut up the 2D space into boxes with length $n^{-1/4}$, average in each box

What’s our estimation error?

Informally, we have $\sqrt{n}$ boxes, each box gets $\sqrt{n}$ samples.

$$Error \approx \frac{1}{\sqrt{n}} + \text{(other smoothness terms)}$$

In $d$-dimensions, this becomes $Error = n^{-1/d}$ - This means scaling is $y = -\frac{1}{d}x + C$

**Takeaway:** flexible ‘nonparametric’ learning has dimension dependent scaling laws.
Intrinsic dimensionality theory of data scaling laws

In case that was a bit too low-level..

1. Scaling laws arise due to polynomial rates of learning $\frac{1}{n^\alpha}$
2. The slope $\alpha$ is closely connected to the *intrinsic dimensionality* of the data.

Some recent work (Bahri+ 2021) have tried to verify this empirically.
Other advanced data scaling law: distribution shift

Data scaling thus far: how does dataset size relate to performance?

Related question: how does dataset composition affect performance

A: Data composition affects the offset, not the slope.

These ‘distribution shift’ scaling laws can tell us about the importance of collecting diverse data!
Other advanced data scaling laws: fairness + distr. shift

**Data diversity:** can we use scaling laws to understand fairness impacts of data?

**Conjecture:** performance for *minority subgroups* also follow a scaling law

We can use scaling laws to optimize data collection for fairness.

[Rolfe+ 2021]
Recap: data scaling laws

- Remarkably linear relationship between log-data size and log-error
- Holds across domains and models
- Theory understanding: similar to generalization bounds: mean estimation example
- Applications: data collection, fairness.
Scaling laws for model engineering

Now for what I promised at the start: model scaling!

Our motivation: how can we efficiently design huge LMs?
- LSTMs vs Transformers
- Adam vs SGD

How should we allocate our limited resources?
- Train models longer vs train bigger models?
- Collect more data vs get more GPUs?

Scaling laws provide a simple procedure to answer these.
Cross-model: transformers vs LSTMs

**Q:** Are transformers better than LSTMs?

**Brute force way:** spend tens of millions to train a LSTM GPT-3

**Scaling law way:**

[Image of a graph showing the comparison between Transformers and LSTMs. The graph illustrates the test loss against parameters (non-embedding). The graph shows that Transformers outperform LSTMs in terms of test loss for varying numbers of layers and parameter counts.]

[Kaplan+ 2021]
Optimizer choice

What about ADAM vs SGD?

[Note, this is in 2017, so pre-transformers. RHN is recurrent highway nets]
Number of layers

Does depth or width make a huge difference?

- 1 vs 2 layers makes a huge difference.
- More layers have diminishing returns below $10^7$ params.
Side note – scaling laws can sometimes lead us astray

These scaling laws are already used in the design of LMs

![Graphs showing Test Loss vs. Parameters](image)

<table>
<thead>
<tr>
<th>Model</th>
<th>$n_{\text{params}}$</th>
<th>$n_{\text{layers}}$</th>
<th>$d_{\text{model}}$</th>
<th>$n_{\text{heads}}$</th>
<th>$d_{\text{head}}$</th>
<th>$n_{\text{vocab}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPT-3 6.7B</td>
<td>6.7B</td>
<td>32</td>
<td>4096</td>
<td>32</td>
<td>128</td>
<td>50K</td>
</tr>
<tr>
<td>J1-Large</td>
<td>7.5B</td>
<td>32</td>
<td>4096</td>
<td>32</td>
<td>128</td>
<td>256K</td>
</tr>
<tr>
<td>GPT-3 175B</td>
<td>175B</td>
<td>96</td>
<td>12288</td>
<td>96</td>
<td>128</td>
<td>50K</td>
</tr>
<tr>
<td>J1-Jumbo</td>
<td>178B</td>
<td>76</td>
<td>13824</td>
<td>96</td>
<td>144</td>
<td>256K</td>
</tr>
</tbody>
</table>

Table 1: Comparing the architecture of our Jurassic-1 models to their GPT-3 counterparts.

[Levine+ 2021]
Some surprising takeaways

The effect of hyperparameters on big LMs can be predicted *before* training!

- Optimizer choice
- Model depth
- Architecture choice

The scaling law based design procedure.
1. Train a few smaller models
2. Establish a scaling law (e.g. ADAM vs SGD scaling law)
3. Select optimal hyperparam based on the scaling law prediction.
Model size data joint scaling

Q: Do we need more data or bigger models?

Clearly, lots of data is wasted on small models

Joint data-model scaling laws describe how the two relate

From Rosenfeld+ 2020,

\[ \text{Error} = n^{-\alpha} + m^{-\beta} + C \]

From Kaplan+ 2021

\[ \text{Error} = [m^{-\alpha} + n^{-1}]^\beta \]

Provides surprisingly good fits to model-data joint error.
Model-data joint scaling is accurate

From Rosenfeld – fit scaling exponents on small data, small models. Predict rest.

Trading off data size and model size: optimize $n^{-\alpha} + m^{-\beta} + C$ with your costs.
Do we have enough data to feed our models?

From Kaplan:

Fitted training laws suggest 22B token WebText can fit $10^9$ parameters.
Model size should scale as $O(m^{0.74})$
Compute tradeoffs.

Q: what about other resources? Compute vs performance?

For a fixed compute budget...

Big model that’s undertrained vs small model that’s well trained?

Scaling laws tell us: properly undertrained models are better
Compute tradeoffs (2)

Q: as we increase both compute and model size, how should we scale training?

- Huge batches, same number of steps
- Fixed batches, more steps

Good news for data parallel processing (?)
Final detail and remark: ‘effective dimensionality’

We’ve been thinking about ‘parameters’ but not all parameters are equal

Embedding layer parameters don’t behave the same!

Related: recent papers on scaling laws for mixtures of experts.
Scaling laws for models and compute

Log-linearity extends to model parameters and compute!

Lets us set the following based on small models
  - Pick optimizer
  - Pick architecture and model sizes

Also lets us make smart resource tradeoffs
  - Big models vs more data?
Q: Can big language models solve every problem?

We can use scaling laws to answer this!

- For each capability (e.g. question answering)
- Build a scaling law for compute capacity.
- Extrapolate the scaling curve.

Can ‘reasonable’ amounts of compute solve our problems?

Taken from r/programmerhumor
Forecasting question: will we solve the Winograd schema?

Classic AI challenge: Winograd schema

<table>
<thead>
<tr>
<th>Twin sentences</th>
<th>Options (answer)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) a The trophy doesn’t fit into the brown suitcase because it’s too large.</td>
<td>trophy / suitcase</td>
</tr>
<tr>
<td>b The trophy doesn’t fit into the brown suitcase because it’s too small.</td>
<td></td>
</tr>
<tr>
<td>(2) a Ann asked Mary what time the library closes, because she had forgotten.</td>
<td>Ann / Mary</td>
</tr>
<tr>
<td>b Ann asked Mary what time the library closes, but she had forgotten.</td>
<td></td>
</tr>
<tr>
<td>(3) a The tree fell down and crashed through the roof of my house. Now, I have</td>
<td>tree / roof</td>
</tr>
<tr>
<td>to get it removed.</td>
<td></td>
</tr>
<tr>
<td>b The tree fell down and crashed through the roof of my house. Now, I have to</td>
<td>tree / roof</td>
</tr>
<tr>
<td>get it repaired.</td>
<td></td>
</tr>
<tr>
<td>(4) a The lions ate the zebras because they are predators.</td>
<td>lions / zebras</td>
</tr>
<tr>
<td>b The lions ate the zebras because they are meaty.</td>
<td></td>
</tr>
</tbody>
</table>

Current GPT-3 performance after seeing 50 examples: 77%. Can we push this further?
How much more compute for human-level reasoning?

Just extend the line for the scaling law..

If the scaling law holds.. Roughly 64 times more parameters will get us to human-level
Another setting: SAT analogies

<table>
<thead>
<tr>
<th>Context</th>
<th>lull is to trust as</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct Answer</td>
<td>cajole is to compliance</td>
</tr>
<tr>
<td>Incorrect Answer</td>
<td>balk is to fortitude</td>
</tr>
<tr>
<td>Incorrect Answer</td>
<td>betray is to loyalty</td>
</tr>
<tr>
<td>Incorrect Answer</td>
<td>hinder is to destination</td>
</tr>
<tr>
<td>Incorrect Answer</td>
<td>soothe is to passion</td>
</tr>
</tbody>
</table>

**Task:** selecting the correct answer (with highest probability)

**Scaling:** clear linear scaling in log space.
Less optimistic scaling curves

Word in context dataset

<table>
<thead>
<tr>
<th>Label</th>
<th>Target</th>
<th>Context-1</th>
<th>Context-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>bed</td>
<td>There's a lot of trash on the <strong>bed</strong> of the river</td>
<td>I keep a glass of water next to my <strong>bed</strong> when I sleep</td>
</tr>
<tr>
<td>F</td>
<td>land</td>
<td>The pilot managed to <strong>land</strong> the airplane safely</td>
<td>The enemy <strong>landed</strong> several of our aircrafts</td>
</tr>
<tr>
<td>F</td>
<td>justify</td>
<td><strong>Justify</strong> the margins</td>
<td>The end <strong>justifies</strong> the means</td>
</tr>
<tr>
<td>T</td>
<td>beat</td>
<td>We <strong>beat</strong> the competition</td>
<td>Agassi <strong>beat</strong> Becker in the tennis championship</td>
</tr>
</tbody>
</table>

**Scaling**: near-zero. GPT-3 paper notes ‘pairwise comparison’ tasks are harder.
Phase transitions

**Thus far:** everything has had linear scaling (with different slopes).

**Phase transitions** are sudden, discontinuous jumps in performance.

The GPT-3 paper has some intriguing observations on phase transitions.

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Do we expect to see more phase transitions? This is probably the ‘big unknown’ in LM scaling!
Scaling laws and the future

Some tasks will just improve continually via scale (Winograd, SAT etc)

There are some others that may have ‘phase transitions’ and emergent behavior

Finally, more work to be done on some tasks (WiC?)

**Scaling laws can help with** a key question: what problems can we brute force?
Recap: scaling laws – surprising and useful!

- **Data scaling**: understand how data affects models, clean theory
- **Model scaling**: dramatically reduce costs for training
- **Scaling as prediction**: understand what problems can be ‘brute forced’

**Scaling laws are interesting for everyone!**

- Theorists (why do we get scaling laws)
- Practitioners (lets use scaling laws to optimize)
- AI enthusiasts (can we get AGI with more gpus?)